Gravitational forces from Bose-Einstein condensation

M. Consoli

Istituto Nazionale di Fisica Nucleare, Sezione di Catania
Corso Italia 57, 95129 Catania, Italy

Abstract

The basic idea that gravity can be a long-wavelength effect *induced* by the peculiar ground state of an underlying quantum field theory leads to consider the implications of spontaneous symmetry breaking through an elementary scalar field. We point out that Bose-Einstein condensation implies the existence of long-range order and of a gap-less mode of the (singlet) Higgs-field. This gives rise to a $1/r$ potential and couples with infinitesimal strength to the inertial mass of known particles. If this is interpreted as the origin of Newtonian gravity one finds a natural solution of the hierarchy problem. As in any theory incorporating the Equivalence Principle, the classical tests in weak gravitational fields are fulfilled as in general relativity. On the other hand, our picture suggests that Einstein general relativity may represent the weak field approximation of a theory generated from flat space with a sequence of conformal transformations. This explains naturally the absence of a *large* cosmological constant from symmetry breaking. Finally, one also predicts new phenomena that have no counterpart in Einstein theory such as typical 'fifth force' deviations below the centimeter scale or further modifications at distances 10^{17} cm in connection with the Pioneer anomaly and the mass discrepancy in galactic systems.

1. Introduction

The basic idea that gravity is a semi-classical, long-wavelength effect *induced* by an underlying quantum field theory is now more than twenty years old [1, 2, 3]. This approach is very appealing since, in fact, one can get a picture of the world with only *three* elementary interactions and where the origin of the fourth, gravitation, has to be searched in the structure of the vacuum. This idea has been recently re-proposed in ref.[4] where the possible origin of gravity has been traced back to the existence of a gap-less mode of the (singlet) Higgs field. In this paper, we shall address the basic problem again from scratch to make clear the simple physical motivations of the proposal and discuss further possible implications.

In the framework of induced-gravity theories, it is natural to investigate the possible role of spontaneous symmetry breaking. Indeed, in the Standard Electroweak Theory, this sets up the ground state and is the origin of the known particle masses.

What kind of minimal requirements have to be met in order to obtain a consistent phenomenological picture ? One possibility is that, hidden in some corner of the theory, there is a gap-less mode of the (singlet) Higgs field that gives rise to the attractive $1/r$ Newton potential. It turns out that this effect, missed so far, can be deduced from very general properties such as the long-range order associated with Bose-Einstein condensation and the non-relativistic energy spectrum of low-density Bose systems with short-range two-body interactions.

In this scenario, this long-range mode is then coupled in an universal way to the *inertial* mass of the known elementary fermions thus automatically implying that 'inertial mass = gravitational mass'. Moreover, at long distances the Higgs coupling to the fermion masses is renormalized into the coupling to the trace of the energy-momentum tensor, that represents the Lorentz-invariant definition of inertia. In this way, one can understand the origin of the Newton constant G out of a theory that, apparently, has only one dimensionful quantity, namely the Fermi constant G_F, thus obtaining a natural solution of the so called 'hierarchy' problem.

Finally, for weak gravitational fields, the classical tests of general relativity would actually be fulfilled [5] in any theory that incorporates the Equivalence Principle and do not necessarily require an underlying *fundamental* tensor theory. While this last remark is essential for the consistency of any theoretical framework with well known experimental results, one also predicts new phenomena that have no counterpart in Einstein theory. For instance, the Newton $1/r$ potential turns out to be modified below the centimeter scale,

with possibly important consequences for the gravitational clustering of matter.

If, on one hand, the very accurate equality between the inertial and gravitational mass of known particles makes, by itself, extremely natural the idea of a 'Higgs-gravity connection' [6] to a closer inspection a tight link between the physical origin of gravity and the physical origin of inertia is also suggested by classical general relativity. Only in this case, in fact, one can understand Einstein's formulation of the 'Mach Principle', namely the consistent vanishing of inertia if gravity would be switched off [7]. We shall return to this important point in the following.

The plane of the paper is as follows. In Sect.2 we shall first review the general features of the excitation spectrum in 4-dimensional quantum field theories that possess a non-trivial vacuum. In Sect.3 we shall present the basic ingredients of spontaneous symmetry breaking in $\lambda\Phi^4$ theories. In Sect.4 we shall discuss the origin of the gap-less mode of the Higgs field. This will be shown in Sect.5 to give rise to the Newton potential. After, in Sect.6 we shall discuss the connections with Einstein general relativity in weak gravitational fields. Finally, we shall present in Sect.7 the summary of our results together with some speculations on further possible implications at the astronomical level.

2. The vacuum and its excitation spectrum

To introduce gravity, some type of deviation from *exact* Lorentz-covariance has to be introduced in order not to run into self-contradictory statements [8]. As anticipated, our main point is that this type of deviation is found in the long-wavelength excitation spectrum of the Higgs field in the spontaneously broken phase. Before addressing any specific detail, let us consider the more general aspects related to our proposal. For instance, the nature of the ground state may lead to violations of causality.

Quite independently of any application to gravity, the possible departure from an exactly Lorentz-invariant vacuum was considered by Segal [9] as a general feature of 4-dimensional non-linear quantum field theories, such as $(\lambda\Phi^4)_4$. The connection with causality can be easily understood since the usual normal-ordering procedure guarantees the local commutativity of Wick-ordered products of the field operator in the free theory. However, no such a procedure is known *a priori* for the interacting case. Thus the argument is circular since the proper normal-ordering procedure is only known *after* determining the vacuum and its excitation spectrum. For actual calculations, one uses the normal-ordering definition of free-field theory and introduces an ultraviolet cutoff Λ or a lattice spacing $r_o \sim 1/\Lambda$ for the remaining divergences [10]. In this approach, where

the continuum theory is defined for $\Lambda \to \infty$, consistency requires that the physical spectrum should approach a Lorentz-covariant form $\sqrt{k^2 + M^2}$ and causality be recovered. On the other hand, for *finite* Λ, however large, one is faced with deviations from a Lorentz-covariant energy spectrum and violations of causality.

A possible objection is that this conclusion reflects the use of a non-Lorentz-invariant ultraviolet regulator. For instance, by using dimensional regularization, where the continuum limit is $d \to 4$, such problems should not arise. This is not so obvious since, up to now, dimensional regularization is known as an essentially perturbative procedure that, indeed, is extremely useful in those situation where a perturbative picture is known to work. After all, this is the reason why one pays so much attention to the results of lattice simulations performed with toy-actions that only asymptotically possess the same symmetry properties of their continuum versions. At the same time, beyond perturbation theory and just in the case of $\lambda\Phi^4$ theories, it is known that the limit $d \to 4$ is ambiguous [11] depending whether $d = 4 - \epsilon$ or $d = 4 + \epsilon$ (for $\epsilon > 0$). Outside of the perturbative domain, similar type of problems can arise in any theory depending on the given trajectory chosen in the complex plane to approach the value $d = 4$.

The problem of the excitation spectrum becomes unavoidable, however, if one starts to model the world as a cutoff-regulated quantum field theory since, in this case, the cutoff will never be removed. However, our point of view, namely that *all* departures from exact Lorentz-covariance are due to gravitational interactions, offers a physical interpretation of the deviations. At the same time, gravity is an extremely weak interaction so that all violations of causality in gravitational fields should be very difficult to observe in ordinary conditions. One can also reverse the argument: if gravity is generated by the vacuum structure of a quantum field theory and causality turns out to be effectively preserved, this means that gravitational effects *cannot* become too strong. At the same time, if we are dealing with the same physical theory, we would expect the problem of causality to occur in general relativity as well. This is precisely what happens since, regardless of the quantum phenomena that give rise to the ground state, it is known that constant energy-density solutions of Einstein equations contain indeed closed time-like curves [12].

Finally, any description of gravity should provide an explanation of the Equivalence Principle. If Einstein theory is considered the fundamental description of gravity, this has the role of a ('philosophical' [13]) principle. On the other hand, if Newtonian gravity is generated by the vacuum structure of a quantum field theory, it is a dynamical consequence and represents the weak-field remnant of an otherwise exactly Lorentz-covariant theory.

We shall return to this important point in Sect. 6.

3. Spontaneous symmetry breaking in $\lambda\Phi^4$ theories

Before addressing the problem of the energy spectrum of spontaneously broken $\lambda\Phi^4$ theories , we have first to consider those general properties of the phase transition that are essential for any further analysis.

The 'condensation' of a scalar field, i.e. the transition from a symmetric phase where $\langle\Phi\rangle = 0$ to the physical vacuum where $\langle\Phi\rangle \neq 0$, has been traditionally described as an essentially classical phenomenon (with perturbative quantum corrections). In this picture, one uses a classical potential

$$V_{\rm cl}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \qquad (3.1)$$

where the phase transition, as one varies the m^2 parameter, is second order and occurs at $m^2 = 0$.

As discussed in ref.[14], the question of vacuum stability is more subtle in the quantum theory. Here, the starting point is the Hamiltonian operator

$$H =: \int d^3x \left[\frac{1}{2}\left(\Pi^2 + (\boldsymbol{\nabla}\Phi)^2 + m^2\Phi^2\right) + \frac{\lambda}{4!}\Phi^4\right] : \qquad (3.2)$$

after quantizing the scalar field Φ and the canonical momentum Π in terms of annihilation and creation operators $a_{\mathbf{k}}$, $a^\dagger_{\mathbf{k}}$ of a reference vacuum state $|o\rangle$ ($a_{\mathbf{k}}|o\rangle = \langle o|a^\dagger_{\mathbf{k}}$=0). These satisfy the commutation relations

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'}. \qquad (3.3)$$

and, due to normal ordering, the quadratic part of the Hamiltonian has the usual form ($E_k = \sqrt{k^2 + m^2}$)

$$H_2 = \sum_{\mathbf{k}} E_k a^\dagger_{\mathbf{k}} a_{\mathbf{k}}. \qquad (3.4)$$

for the elementary quanta of the symmetric phase ('phions').

Now the trivial vacuum $|o\rangle$ where $\langle\Phi\rangle = 0$ is clearly locally stable if phions have a physical mass $m^2 > 0$. However, is an $m^2 > 0$ symmetric vacuum necessarily *globally* stable ? Could the phase transition actually be first order, occurring at some small but positive value of the physical mass squared $m^2 > 0$? The question is not entirely trivial just because [14] the static limit of the 2-body phion-phion interaction is not always repulsive. Besides the tree-level repulsive potential there is an induced attraction from higher-order

graphs. In this case, for sufficiently small values of m, the trivial 'empty' state $|o\rangle$ may not be the physical vacuum.

The answer to the question depends on the form of the *effective potential* $V_{\rm eff}(\phi)$ and it is not surprising that different approximations may lead to contradictory results on this crucial issue. The situation is similar to the Bose-Einstein condensation in condensed matter that is a first-order phase transition in an ideal gas. However, in interacting systems the issue is more delicate and often difficult to be settled experimentally. Theoretically is predicted to be a second-order transition in some approximations but it may appear as a weak first-order transition in other approximations [15].

We shall refer to [14, 16] for details on the structure and the meaning of various types of approximations to the effective potential and just report a few basic results:

i) the phase transition is indeed first order as in the case of the simple one-loop potential. This is easy to realize if one performs a variational procedure, within a simple class of trial states that includes $|o\rangle$. In this case, one finds [17] that the $m = 0$ theory lies in the broken phase. Therefore the phase transition occurs earlier, for some value of the phion mass $m \equiv m_c$ that is still positive. This conclusion is confirmed by the results of ref.[18] that provides the most accurate non-perturbative calculation of the effective potential of $\lambda\Phi^4$ theories performed so far.

Understanding the magnitude of m_c requires additional comments. As recalled in Sect.2, the normal ordering prescription in Eq.(3.2) eliminates all ultraviolet divergences of the free-field case at $\lambda = 0$. However, for $\lambda > 0$ there are additional divergences. For this reason, one introduces an ultraviolet cutoff Λ and defines the continuum theory as a suitable limit $\Lambda \to \infty$. In this case, however, one is faced with a dilemma since a meaningful description of SSB in quantum field theory *must* provide $m_c = 0$. Otherwise, from the existence of a non-vanishing mass gap controlling the exponential decay of the two-point function of the symmetric phase, and the basic axioms of quantum field theory [19] one would deduce the uniqueness of the vacuum (and, thus, no SSB). The resolution of this apparent conflict [16, 20, 14] is that the continuum limit of the cutoff-regulated theory gives a vanishing ratio ($\epsilon \equiv \frac{1}{\ln \frac{\Lambda}{M_h}}$)

$$\frac{m_c^2}{M_h^2} \sim \epsilon \tag{3.5}$$

so that when the scale of the spontaneously broken phase, namely the Higgs boson mass M_h, is taken as the unit scale of mass, the possible values of the phion mass $0 \leq m \leq m_c$ are naturally infinitesimal. In this sense, SSB is an *infinitesimally weak* first-order phase

transition where the magnitude of the ratio $\frac{m}{M_h}$ represents a measure of the degree of non-locality of the cutoff-regulated theory.

ii) there is a deep difference between a 'free-field' theory and a 'trivial' theory [21] where the interaction effects die out in the continuum limit. The former has a quadratic effective potential and a unique ground state. The latter, even for a vanishingly small strength $\lambda = \mathcal{O}(\epsilon)$ of the elementary two-body processes can generate a *finite* gain in the energy density, and thus SSB, due to the macroscopic occupation of the same quantum state, namely to the phenomenon of Bose condensation. This leads to a large re-scaling of $\langle \Phi \rangle$. Indeed, one can introduce, in general, two distinct normalizations for the vacuum field ϕ, say a 'bare' field $\phi = \phi_B$ and a 'renormalized' field $\phi = \phi_R$. They are defined through the quadratic shapes of the effective potential in the symmetric and broken phase respectively

$$\left.\frac{d^2 V_{\rm eff}}{d\phi_B^2}\right|_{\phi_B=0} \equiv m^2, \qquad \left.\frac{d^2 V_{\rm eff}}{d\phi_R^2}\right|_{\phi_R=v_R} \equiv M_h^2. \tag{3.6}$$

Due to 'triviality', the theory is "nearly" a massless, free theory so that $V_{\rm eff}$ is an extremely flat function of ϕ_B. Therefore, due to (3.5), the re-scaling Z_ϕ relating ϕ_B and ϕ_R becomes very large. By defining $\phi_B^2 = Z_\phi \phi_R^2$, one finds $Z_\phi = \mathcal{O}(\frac{1}{\epsilon})$ or

$$v_R \sim v_B \sqrt{\epsilon} \tag{3.7}$$

Just for this reason, the rescaling of the 'condensate' $Z = Z_\phi$ is different from the more conventional quantity $Z = Z_{\rm prop}$ defined from the residue of the shifted field propagator at $p^2 = M_h^2$. According to Källen-Lehmann decomposition and 'triviality' this has a continuum limit $Z_{\rm prop} = 1 + \mathcal{O}(\epsilon)$.

iii) the existence of two different continuum limits $Z_\phi \to \infty$ and $Z_{\rm prop} \to 1$ reflects a fundamental discontinuity in the 2-point function at $p = 0$ (p= Euclidean 4-vector). This effect is not totally unexpected and its origin should be searched in the infrared divergences of perturbation theory for 1PI vertices at zero external momenta [22]. Of course, after the Coleman-Weinberg [23] analysis, we know how to obtain infrared-finite expressions for 1PI vertices at zero external momenta. This involves summing up an infinite series of graphs of different perturbative order with different numbers of external legs, just as in the analysis of the effective potential that was taken as the starting point for our analysis. In this case, the second derivative of the effective potential gives $\Gamma^{(2)}(p = 0)$, the inverse susceptibility

$$\chi^{-1} = \left.\frac{d^2 V_{\rm eff}}{d\phi_B^2}\right|_{\phi_B=v_B} = \frac{M_h^2}{Z_\phi} \tag{3.8}$$

Therefore, if $Z_\phi = \mathcal{O}(\frac{1}{\epsilon})$, one finds

$$\frac{\Gamma^{(2)}(0)}{M_h^2} \sim \epsilon \tag{3.9}$$

rather than $\Gamma^{(2)}(0) = M_h^2$ as expected for a free-field theory where

$$\Gamma^{(2)}(p) = (p^2 + M_h^2) \tag{3.10}$$

Notice that the discrepancy found in the discrete-symmetry case implies the same effect for the zero-momentum susceptibility of the *radial* field in an O(N) continuous-symmetry theory. This conclusion, besides the general arguments of [22], is supported by the explicit calculations of Anishetty et al [24].

Notice that SSB requires the subtraction of disconnected pieces so that continuity at $p = 0$ does not hold, in general [25]. At the same time, a mismatch at $p = 0$ does not violate 'triviality' since no scattering experiment can be performed with exactly zero-momentum particles. On the other hand, for large but finite values of the ultraviolet cutoff Λ, when 'triviality' is not complete, the discrepancy between $\Gamma^{(2)}(0)$ and M_h^2 will likely 'spill over' into the low-momentum region $p^2 \sim \epsilon M_h^2$. In this region, we expect sizeable differences from the free-field form Eq.(3.10).

If really $Z_\phi \neq Z_{\text{prop}}$ this result has to show up in sufficiently precise numerical simulations of the broken phase. To this end, the structure of the two-point function has been probed in refs. [26] by using the largest lattices considered so far. One finds substantial deviations from Eq.(3.10) in the low-p region and only for large enough p, $\Gamma^{(2)}(p)$ approaches the free field form (3.10). Also, the lattice data of refs.[26] support the prediction that the discrepancy between $\Gamma^{(2)}(0)$ and the asymptotic value M_h^2 becomes larger when approaching the continuum limit.

We stress that no such a discrepancy is present in the symmetric phase where $\langle\Phi\rangle = 0$. Here the free-field behaviour $\Gamma^{(2)}(p) = (p^2 + m^2)$ is valid to high accuracy down to $p = 0$ [26]. Notice the different effect of the cutoff in the broken and symmetric phases. In both cases, the limit $\Lambda \to \infty$ yields a free spectrum of the type (3.10). In the broken phase, however, this is obtained by making sharper and sharper a discrepancy at low k so that a discontinuity at $k = 0$ will remain.

In conclusion: theoretical arguments and numerical evidences suggest that in the limit $k \to 0$ the excitation spectrum of the broken phase can show substantial deviations from the free-field form $\tilde{E} = \sqrt{k^2 + M_h^2}$. Due to the 'triviality' of the theory, the deviations from the free-field behaviour should, however, be confined to a range of k that becomes infinitesimal in units of M_h in the continuum limit $\Lambda \to \infty$.

4. A gap-less mode of the Higgs field: the vacuum is not 'empty'

In this section we shall present some very general arguments to illustrate the nature of the excitation spectrum of the broken phase in the limit $k \to 0$. The discussion is valid in the framework of a weakly first-order phase transition where one can meaningfully describe the broken phase as a Bose condensate of the elementary quanta of the symmetric phase.

The starting point for our analysis is a positive-definite (but otherwise arbitrary) relation between the number density n of condensed phions at $k = 0$ and the scalar field expectation value, namely

$$n = n(\phi_B) \tag{4.1}$$

Using Eq.(4.1) one can easily transform the energy density $\mathcal{E} = \mathcal{E}(n)$ into the effective potential $V_{\text{eff}} = V_{\text{eff}}(\phi_B)$. In this way, the $\phi_B = 0$ 'mass-renormalization' condition in (3.6)

$$\left.\frac{d^2V_{\text{eff}}}{d\phi_B^2}\right|_{\phi_B=0} \equiv m^2 \tag{4.2}$$

becomes

$$\left.\frac{\partial\mathcal{E}}{\partial n}\right|_{n=0} = m. \tag{4.3}$$

Its physical meaning is transparent. If we consider the symmetric vacuum state ("empty box") and add a very small density n of phions (each with vanishingly small 3-momentum $k \to 0$) the energy density is $\mathcal{E}(n) - \mathcal{E}(0) \sim nm$ in the limit $n \to 0$.

Let us now analyze spontaneous symmetry breaking. This can be viewed as a phion-condensation process occuring at those values $\phi_B = \pm v_B$ where

$$\left.\frac{dV_{\text{eff}}}{d\phi_B}\right|_{\phi_B=v_B} = 0 \tag{4.4}$$

By using Eq.(4.1) and defining the ground-state particle density

$$n_v = n(v_B) \tag{4.5}$$

we also obtain

$$\left.\frac{\partial\mathcal{E}}{\partial n}\right|_{n=n_v} = 0 \tag{4.6}$$

Eq.(4.6), differently from Eq.(4.3), means that small changes of the phion density around its stationarity value do not produce any change in the energy density of the system. Namely, $\mathcal{E}(n) - \mathcal{E}(n_v) \sim (n - n_v)^2$ and, as a consequence of condensation, one can add or remove an arbitrary number of phions at $k = 0$ without any energy cost, just as in the non-relativistic limit of the theory.

Therefore, for $k \to 0$, the excitation spectrum of the theory exhibits the following features:

a) <u>in the symmetric phase</u> $\qquad E(k) \sim m + \frac{k^2}{2m} \to m$

(which is the standard spectrum for massive particles)

b) <u>in the broken phase</u> $\qquad \tilde{E}(k) \to 0$

(which is the condition of a gap-less spectrum)

In this sense, in the broken phase, the $k \to 0$ Fourier component of the scalar field behaves as a massless field. We now understand why, in the broken phase, the excitation spectrum $\tilde{E}$ cannot be $\sqrt{k^2 + M_h^2}$ when $k \to 0$: this form does not reproduce $\tilde{E} = 0$ at $k = 0$.

One may object that the excitation spectrum is *discontinuous* so that one has exactly $\tilde{E} = \sqrt{k^2 + M_h^2}$ for *all* $k \neq 0$ except at $k = 0$. First of all, this is not what is generally believed since the rest mass M_h is generally identified with the energy-gap $\tilde{E}(k = 0)$ of the broken phase (at least in the discrete-symmetry case where there are no Goldstone bosons). Moreover, as anticipated in Sect.3, this type of behaviour is, indeed, expected for the *continuum* theory. In a cutoff theory, however large Λ may be, all singularities are smoothed and one has a continuous spectrum for all values of k. This point of view is also consistent with the sequence of lattice calculations of ref.[26].

The existence of the gap-less mode is directly related to Bose-Einstein condensation [15, 27]. Indeed, the behaviour of the spectrum for $k \to 0$ in the broken phase corresponds to the range of momenta $k \ll m$ of the *non-relativistic* theory. In this regime a scalar condensate, whatever its origin may be, is a highly correlated structure with *long-range* order. This is clear from the following general argument [15] due to Anderson. Suppose that in a large box of volume $\mathsf{V} \to \infty$ we have a condensate of $\mathsf{N} \to \infty$ particles in the $k = 0$ mode. Let us divide the box into a large number of K identical and macroscopic subsystems so that each subsystem still contains a very large number (N/K) of particles. Let us also denote A_i the annihilation operator for the $k = 0$ particles contained in the ith subsystem. In this case, we have $\langle a_o^\dagger a_o \rangle = \mathsf{N}$, $\langle A_i^\dagger A_i \rangle = \mathsf{N}/K$, so that from

$$a_o = \frac{1}{\sqrt{K}} \sum_i A_i \tag{4.7}$$

we obtain

$$\mathsf{N} = \frac{\mathsf{N}}{K} + \frac{1}{K} \sum_{i \neq j} \langle A_i^\dagger A_j \rangle \tag{4.8}$$

For large K, the second term must dominate so that, the phases of the A_i in different subsystems must be correlated. In the limit where $K \to \infty$ (but still N/K is a large number) this is equivalent to introduce a *complex* condensate wave-function at each point in space $\Psi \sim \sqrt{n}e^{i\theta}$ that represents the true order parameter to describe the response of the condensate to the very long wavelengths with $k \ll m$. Although the energy density does not depend on the possible constant values of the phase, the vacuum state will pick up just one of them. In this sense, the gap-less mode of the Higgs field can be considered the Goldstone boson of a spontaneously broken *continuous* symmetry, the phase rotations of the non-relativistic wave-function of the condensate that does not exist in the symmetric phase.

Therefore, even in the case of spontaneous symmetry breaking with a *neutral* scalar field, the ground state is still infinitely degenerate [29] and adding particles with $k \to 0$ will only induce an energy density $\sim (\nabla\theta)^2$. Truly enough, this effect shows up only for very small values of k and is not perceivable at short distances. Just for this reason, the peculiar phenomenon at the basis of the gap-less mode of the Higgs field has nothing to do with the Goldstone bosons that give rise to the W and Z masses. These arise from the spontaneous breaking of continuous symmetries that are already seen in the symmetric phase (where there are no condensates whatsoever).

Notice that our result, although deduced within the framework of ref.[14], does not depend on the validity of the relation

$$n \sim \tfrac{1}{2}m\phi_B^2, \tag{4.9}$$

used in ref.[14]. Indeed, Eq.(4.6) follows from Eq.(4.4) *regardless* of the precise functional relation between the phion density n and the vacuum field ϕ_B. Moreover, the same conclusions hold in any description based on a first-order phase transition where the broken phase can be represented as a condensate of the elementary quanta of the symmetric phase. For instance, the phase transition remains first-order if spontaneous symmetry breaking is induced by (or contains the additional contributions of) intermediate vector bosons [23, 30]. In this case, as in pure $\lambda\Phi^4$ theory, the massless theory at $m = 0$ is found in the broken phase so that the phase transition occurs earlier at a non-zero and positive m_c^2.

After this general discussion, let us now attempt a semi-quantitative description of the energy spectrum of the broken phase. A first observation is that for $k \sim m$ (or larger) we expect

$$\tilde{E}(k) \sim \sqrt{k^2 + M_h^2} \tag{4.10}$$

On the other hand, the region $k \to 0$ of *low-density* Bose systems, with short-range 2-body interactions, can be analyzed in a universal way [31] namely

$$\tilde{E} \sim c_s k \qquad \text{for } k \to 0 \tag{4.11}$$

where c_s is the sound velocity. In a simple picture, the two branches of the spectrum join through some form of continuous matching at momenta $k \sim m$ (see fig.1), analogously to the case of 'phonons' and 'rotons' in superfluid He^4. If we recall that $M_h >> m$, we then obtain the order of magnitude estimate

$$c_s \sim \frac{M_h}{m} \tag{4.12}$$

This result can be expressed in a more quantitative form if one uses the precise relation for dilute Bose systems [32]

$$c_s \equiv \frac{1}{m}\sqrt{4\pi n a} \tag{4.13}$$

in terms of the physical S-wave 'phion-phion' scattering length a. In this case, by using the results of ref.[14], we find $a \sim \frac{\lambda}{8\pi m}$ and the expression for the Higgs mass [14]

$$M_h^2 \equiv 8\pi n a \tag{4.14}$$

so that we get the final result

$$c_s = \frac{M_h}{m\sqrt{2}} \equiv \sqrt{\eta} \tag{4.15}$$

Notice that Eq.(4.15) makes no reference to the bare coupling λ entering the hamiltonian density (3.2) and would be formally unchanged if the scalar self-interaction were replaced by a short-range interaction that includes the effect of vector-boson and/or fermion loops [23].

Eqs. (4.11) and (4.13) become a better and better approximation in the limit of very low-densities $na^3 \to 0$ where all condensed phions are found in the state at $k = 0$ and there is no population of the finite momentum modes ('depletion') since

$$D \equiv 1 - \frac{\mathsf{N}(k=0)}{\mathsf{N}} = \mathcal{O}(\sqrt{na^3}) \tag{4.16}$$

The depletion is a simple phase-space effect representing the probability that, besides the condensate, also states such as $(\mathbf{k}, -\mathbf{k})$ are populated in the ground state. It represents a measure of interaction effects that cannot be re-absorbed into the linear energy spectrum [32] and, therefore, can be viewed as *residual* interaction. In this respect, spontaneous symmetry breaking in a cutoff $\lambda\Phi^4$ theory corresponds to the case of an *almost* ideal, dilute

Bose system. In fact, 'triviality' requires a continuum limit with a vanishing strength $\lambda = \mathcal{O}(\epsilon)$ for the elementary 2-body processes. Together with Eq.(3.5), this leads to $aM_h \sim \sqrt{\epsilon}$. Therefore, by taking $M_h^2 \equiv 8\pi na$ as the physical scale of the theory in the broken phase, we find a continuum limit where $a \to 0$, $n \to \infty$ with $na = const.$ and

$$na^3 = \mathcal{O}(\epsilon) \tag{4.17}$$

When $\epsilon \to 0$, the phion-condensate becomes infinitely dilute so that the average spacing between two phions in the condensate, $d \equiv n^{-1/3}$, becomes enormously larger than their scattering length. In this limit, the energy spectrum (4.11) becomes exact (for $k \to 0$) while, for finite Λ, there are $\mathcal{O}(\epsilon)$ corrections and a small, but finite, depletion with density

$$\frac{n_D}{n} = \mathcal{O}(\sqrt{\epsilon}) \tag{4.18}$$

Notice, however, that the phion density $n \sim \frac{1}{2}mv_B^2$, is very large, $\mathcal{O}(\epsilon^{-1/2})$, in the physical units denoted by the correlation length $\xi_h \equiv 1/M_h$. Indeed, $\frac{d}{\xi_h} \sim \epsilon^{1/6}$. It is because there is such a high density of phions that their tiny 2-body interactions $\mathcal{O}(\epsilon)$ can produce a finite effect on the energy density. In this sense, the phion condensate is a very dilute gas when observed on the very small scale of the phion-phion scattering length a but may appear as a very dense *liquid* on larger scales.

At the same time, Bose liquids at zero-temperature are known to possess the remarkable property of *superfluidity* so that the scalar condensate, when placed in an external field, flows without friction. For this reason, the result $\tilde{E}(k) \sim c_s k$ for $k \ll m$, deduced from the quantum dynamics of weakly coupled Bose systems with short-range two-body interactions, could have been obtained by requiring a frictionless motion of macroscopic bodies in the vacuum. Namely, the condition that the scalar condensate cannot absorb arbitrarily small amounts of energy-momentum transfer, for $k \ll m$, is precisely the starting point used by Landau [33] to deduce the linear excitation spectrum of a superfluid at low k where the motion is frictionless provided the velocity of an external body is $|\mathbf{u_e}| \leq c_s$. In the case of the phion condensate, this is not a restriction in view of the fantastically high value of the 'sound-velocity' $c_s \sim \frac{M_h}{m}\ c >> c$ [34]. At the same time, the residual self-interaction effects embodied in the presence of a non-zero depletion can give rise to a small friction when studying the superfluid flow over very large distances.

In conclusion: spontaneous symmetry breaking in a cutoff $\lambda\Phi^4$ theory gives rise to an excitation spectrum that is *not* exactly Lorentz-covariant. The usual assumption $\tilde{E}(k) \sim \sqrt{k^2 + M_h^2}$ is not valid in the limit $k \to 0$ where one actually finds a 'sound-wave' shape $\tilde{E}(k) \sim c_s k$. This result reflects the *physical* presence of the scalar condensate. However, as

expected from our analysis in Sect.3, all deviations from a free-field spectrum are confined to a range of momenta $k \ll m$ that becomes infinitesimal, in units of M_h, in the limit $\Lambda \to \infty$.

5. A long-range potential in Higgs condensates

It is well known that condensed-matter systems can support long-range forces even if the elementary constituents have only short-range 2-body interactions. Just for this reason, it is not surprising that the existence of a gap-less mode for $k \to 0$ in the broken phase can give rise to a long-range potential. For instance, when coupling fermions to a (real) scalar Higgs field with vacuum expectation value v through the Standard Model interaction term

$$- m_i \bar{\psi}_i \psi_i (1 + \frac{h(x)}{v}) \tag{5.1}$$

the static limit $\omega \to 0$ of the scalar propagator

$$D(k, \omega) = \frac{1}{\tilde{E}^2(k) - \omega^2 - i0^+} \tag{5.2}$$

gives rise to an attractive potential between any pair of masses m_i amd m_j

$$U(r) = -\frac{m_i m_j}{v^2} \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{\tilde{E}^2(k)} \tag{5.3}$$

By assuming Eqs.(4.10) and (4.11) for $k \to \infty$ and $k \to 0$, and using the Riemann-Lebesgue theorem [35] on Fourier transforms, the leading $r \to \infty$ behaviour is universal. Any form of the spectrum that for $k \sim m$ interpolates between the two asymptotic trends would produce the same result. At large distances $r >> 1/m$ one finds ($\eta = \mathcal{O}(\frac{1}{\epsilon})$)

$$U(r) = -\frac{G_F}{4\pi\eta} \frac{m_i m_j}{r} [1 + \mathcal{O}(1/mr)] \tag{5.4}$$

where $G_F \equiv 1/(v^2)$. In the physical case of the Standard Model one would identify $G_F \sim 1.1664 \cdot 10^{-5}$ GeV^{-2} with the Fermi constant.

Notice that the coupling in Eq.(5.1) naturally defines the 'Higgs charge' of a given fermion as its physical mass. However, for nucleons, this originates from more elementary Higgs-quark and quark-gluon interactions. At low k these effects can be resummed to all orders by replacing Eq.(5.1) with the effective coupling to the trace of the energy-momentum tensor T^μ_μ. Namely, by denoting $\tilde{h}$ the long-wavelength of the Higgs field

associated with the linear part of the spectrum, we can write down the first few terms of an effective lagrangian ($\tilde{\phi} \equiv \frac{\tilde{h}(x)}{v}$)

$$\mathcal{L}(\tilde{\phi}) = \frac{v^2}{2}\tilde{\phi}[\eta\nabla^2 - \frac{\partial^2}{c^2\partial t^2}]\tilde{\phi} - T^\mu_\mu(1+\tilde{\phi}) + ... \tag{5.5}$$

In Eq.(5.5) the dots indicate cubic and higher order terms describing residual self-interaction effects of the type discussed in Sect.4 and we have made explicit the factor η coming from the peculiar nature of the energy spectrum $\tilde{E}(k) = \sqrt{\eta}k$ for $k \to 0$. The different normalization of the linear coupling reduces to the usual definition in the case of free fermions and yields exactly the nucleon mass when evaluating the matrix element between nucleon states

$$\langle N|T^\mu_\mu|N\rangle = m_N \bar{\psi}_N \psi_N \tag{5.6}$$

We note that the strength of the long-range potential is proportional to the product of the masses and is naturally infinitesimal in units of G_F. It would vanish in a true continuum theory where the gap-less mode of the Higgs field disappears and $\eta \to \infty$. Therefore, it is natural to relate this extremely weak interaction to the gravitational potential and to the Newton constant G by identifying

$$\sqrt{\eta} = \sqrt{\frac{G_F}{G}} \sim 10^{17} \tag{5.7}$$

Notice that Eq.(5.5) rensembles a Brans-Dicke theory [36]. Here, however, the framework is very different since the $\tilde{\phi}-$field propagates in the presence of the phion condensate. Approximating the field $\tilde{\phi}$ as a free field with $\tilde{E}^2(k) = \eta k^2$, we can write down its equation of motion, namely

$$[\eta\nabla^2 - \frac{\partial^2}{c^2\partial t^2}]\tilde{\phi} = \frac{T^\mu_\mu}{v^2} \tag{5.8}$$

that, due to the fantastically high value of η, reduces for all practical applications to

$$\nabla^2\tilde{\phi} = GT^\mu_\mu \tag{5.9}$$

Finally, for classical motions in the limit of velocities $|\mathbf{u}_n| \ll c$, when the trace of the energy-momentum tensor [37]

$$T^\mu_\mu(x) \equiv \sum_n \frac{E_n^2 - \mathbf{p}_n \cdot \mathbf{p}_n}{E_n}\delta^3(\mathbf{x} - \mathbf{x}_n(t)) \tag{5.10}$$

reduces to the mass density

$$\sigma(x) \equiv \sum_n m_n \delta^3(\mathbf{x} - \mathbf{x}_n(t)) \tag{5.11}$$

Eq.(5.9) becomes the Poisson equation for the Newton potential

$$\nabla^2\tilde{\phi} = G\sigma(x) \tag{5.12}$$

Notice that the long-range $1/r$ potential is a direct consequence of the existence of the scalar condensate. Therefore, speaking of gravitational interactions makes sense only for particles that can induce variations of the phion density by exciting the gap-less mode of the Higgs field. In this sense, phions, although possessing an inertial mass, have no 'gravitational mass'.

In the physical case of the Standard Model, and assuming the range of Higgs mass $M_h \sim 10^2 - 10^3$ GeV, we obtain a range of phion masses $m \sim 10^{-4} - 10^{-5}$ eV. The detailed knowledge of the spectrum $\tilde{E}(k)$ for $k \sim m$ would allow to compute the terms $\mathcal{O}(1/mr)$ in Eq.(5.4) and predict a characteristic pattern of 'fifth force' deviations below the centimeter scale.

As anticipated in Sect.4, the nature of the vacuum implies the scalar condensate to behave as a superfluid at zero temperature. Therefore, its velocity field $\mathbf{u_s}$ corresponds to a potential flow

$$\nabla \ \mathbf{x} \ \mathbf{u_s} = 0 \tag{5.13}$$

This provides a simple hydrodynamical picture of Newtonian gravity. Indeed, by identifying

$$\mathbf{u}_s = \frac{\nabla\tilde{\phi}}{m} \tag{5.14}$$

and introducing an average constant phion density $\langle n\rangle$, the Poisson equation can be rewritten as

$$\nabla\cdot(\langle n\rangle\mathbf{u}_s) = \sigma(x) \tag{5.15}$$

provided we identify

$$\langle n\rangle = \frac{m}{G} \tag{5.16}$$

Eqs.(5.14) and (5.15) establish a formal relation between the difference of the gravitational potential and the associated superfluid flow. In this picture $\tilde{\phi}$ is a classical field determining the phase of the non-relativistic condensate wave function $\Psi \sim \sqrt{\langle n\rangle}e^{i\tilde{\phi}}$ [28].

On the other hand, the Poisson equation (5.12) is modified if the phion density $n \equiv f(x)\langle n\rangle$ sizeably differs from its constant value (5.16) related to the Newton constant. In this case we find instead

$$\nabla\cdot(n\mathbf{u}_s) = \sigma(x) \tag{5.17}$$

Therefore, since the scalar density n can depend on x only through the local gravitational acceleration field, we obtain the modified Poisson equation

$$\nabla \cdot \left[f(\frac{|\nabla \tilde{\phi}|}{g_o}) \nabla \tilde{\phi} \right] = G\sigma(x) \tag{5.18}$$

where a constant acceleration g_o has been introduced to make f dimensionless. The transition from Eq.(5.15) to Eq.(5.17) corresponds to include the effects of residual self-interactions into the corresponding hydrodynamics of a low-temperature Bose liquid (see [28]). In this case, Eq.(5.15) corresponds to the linearized approximation.

Notice that Eq.(5.18) is formally identical to the non-linear modification of inertia ('MOND') introduced by Milgrom [38] to resolve the substantial mass discrepancy and describe many experimental features of galactic systems. This approach represents an alternative to the dark-matter hypothesis and predicts drastic departures from Netwonian dynamics in the typical astronomical large-scale and low-acceleration conditions. These occur when the gravitational acceleration of bodies becomes comparable to a cosmic acceleration field (H is the Hubble constant)

$$g_o \sim cH \sim 10^{-8} \text{ cm sec}^{-2} \tag{5.19}$$

In our picture, this should also correspond to a regime where the phion density n differs substantially from its value in (5.16).

The connection with the Hubble constant can be understood by exploring the implications of Bose-Einstein condensation in an expanding universe. In this case, there must be a continuous creation of phions at a rate

$$\frac{\delta \mathsf{N}}{\mathsf{N}} \sim \frac{\delta \mathsf{V}}{\mathsf{V}} \sim 3H\delta t \tag{5.20}$$

to maintain the same particle density n_v that minimizes the energy density Eqs.(4.4)-(4.6) [39]. This gives rise to a cosmic flow $\langle \mathbf{u}_s \rangle$ that may be used to define a cosmic acceleration field

$$g_o \equiv m|\langle \mathbf{u}_s \rangle| \tag{5.21}$$

that does not depend on the local distribution of gravitational sources. If Hubble expansion takes place only in intergalactic space, the effects of g_o are not observable in solar-system tests. We shall return to this point in the conclusions.

6. Comparison with general relativity in weak gravitational fields

To illustrate the connection with general relativity, we observe preliminarily that Einstein's description of gravity is purely geometric and macroscopic. As such, it does not depend on any hypothesis about the physical origin of this interaction. For instance, classical general relativity, by itself, is unable to predict [7] even the *sign* of the gravitational force (attraction rather than gravitational repulsion). Rather, Einstein had to start from the peculiar properties of Newtonian gravity to get the basic idea of transforming the classical effects of this type of interaction into a metric structure. For this reason, classical general relativity cannot be considered a truly *dynamical* explanation of the origin of the gravitational forces.

It is obvious that in a description where gravity is a long-wavelength excitation of the scalar condensate there are differences with respect to the standard ideas. For instance, the gravitational force is naturally instantaneous. The velocity of light c is quite unrelated to the long-wavelength excitations of the scalar condensate that for $k \to 0$ propagate with the fantastically high speed $c_s = \sqrt{\eta}c \sim 10^{17}c$. Only for $k \sim m$, i.e. at the joining of the two branches of the excitation spectrum, one recovers the expected result $d\tilde{E}/dk < c$. To a closer inspection, this apparently bizzarre result appears less paradoxical than the generally accepted point of view that considers the inertial forces in an accelerated laboratory as the consequence of a gravitational wave generated by distant accelerated matter. Indeed, if the gravitational interaction would really propagate with the light velocity, distant matter must be accelerated *before* the inertial reaction is actually needed [41]. Similar conclusions are also suggested by the analysis of tidal forces [42].

The instantaneous nature of the long-range gravitational interaction is a direct consequence of its non-local origin from the scalar condensate. As anticipated in the Introduction, and on the basis of the hydrodynamical picture of gravity of Sect.5, this leads to a 'Mach's Principle' view of inertia. Indeed, one can imagine that removing at spatial infinity *all* gravitational sources produces an infinite flow of the scalar condensate and,as a net result, the vacuum becomes 'empty' around a given body and its inertia vanishes [43]. Therefore, one cannot speak of absolute accelerations with respect to empty space since the inertial mass of a test particle depends on the existence of the scalar condensate whose density is determined by the distribution of gravitating matter. In this sense, the 'Mach Principle' represents a concise formulation of the inextricable connection between inertia and gravity due to their common origin from the same physical phenomenon: the

condensation of the scalar field . Notice that Mach's ideas had a strong influence on the origin of general relativity [44]. However, to our knowledge, the physical mechanisms for which matter 'there' can determine inertia 'here' had never been addressed.

Finally, one should consider the different impact on general relativity of possible modifications of the Newton potential. A long-distance replacement $\frac{1}{r} \to \frac{\exp(-\mu r)}{r}$ was indeed considered by Einstein [45] in connection with the cosmological problem and the introduction of a cosmological term in the field equations. On the other hand, a modification of the $1/r$ potential below the centimeter scale would imply that classical general relativity is a truly *effective* theory. As such, it would hardly make sense to consider it a 'bare' theory, i.e. the starting point for a quantization procedure. While this point of view is consistent with the induced-gravity approach, where Einstein theory represents, indeed, the weak-field approximation in an all-order expansion in the Riemann tensor [1, 2, 3], one should realize that, if gravitational clustering of matter is modified at short distances, the Schwarzschild singularity may be just an artifact of the approximation.

After this preliminary discussion, let us try to understand whether our description of gravity is consistent with the experimental results. We observe that our picture yields Newtonian gravity and, as such, predicts that bodies with different inertial masses undergo the same acceleration in a given external gravitational field. Therefore, for weak gravitational fields, a freely falling observer can be considered an inertial frame. As anticipated, this is a consequence of starting from an originally Lorentz-invariant theory and where all possible deviations represent just different aspects of the same physical phenomenon: gravitation. Freely falling in weak gravitational fields, is just a way to recover approximate Lorentz-covariance.

For this reason, the basic question about the validity of our picture reduces to the possibility to distinguish between general relativity and *any* theory that incorporates the Equivalence Principle, by performing experiments to an accuracy $\mathcal{O}(G)$. More precisely, is it possible to explain the three classical experimental tests (gravitational red-shift, deflection of light and precession of perihelia) without necessarily introducing the concept of a non-flat metric determined from the energy-momentum tensor by solving the field equations with suitable boundary conditions ? If this is true, classical general relativity cannot be considered the only possible description of gravity.

Now, in the case of a centrally symmetric field both the gravitational red-shift and the *correct* value for the deflection of light were obtained by Schiff [5], long time ago by simple use of the Equivalence Principle and Lorentz transformations. As a consequence,

at the present, only the precession of perihelia can be considered to depend on the full details of Einstein theory, i.e. on the solution of the field equations represented by the Schwarzschild metric

$$ds^2 = c^2 dt^2 \left[1 - \frac{2GM}{c^2 r}\right] - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - dl^2 \tag{6.1}$$

where

$$dl^2 = r^2 \left[d\theta^2 + \sin^2\theta d\varphi^2\right] \tag{6.2}$$

On the other hand, the Equivalence Principle is a weak-field property and for the case of the perihelia the relevant gravitational effects are $\mathcal{O}(10^{-2})$ weaker than for the deflection of light. This suggests that the same technique should also work in this case. Due to the importance of the issue, we shall describe the proof in detail.

Let us start by considering the meaning of Eq.(6.1). In general relativity, this is a solution of the field equations with flat-space boundary conditions at infinity. On the other hand, without specifying the units of length and time dr, dt in (6.1) one cannot understand the physical interpretation of the reference frame where the precession is actually measured [7].

In the case of the gravitational field of a large mass M (e.g the sun) let us consider the set of bound observers $O(i)$'s, freely falling along Keplerian orbits $r \sim r(i)$, and denote their space-time units $(dt(i), dr(i), dl)$. To transfer the informations at spatial infinity, we can use a freely falling observer K_o with zero total energy in the gravitational field. This can be considered as moving with a radial 'escape' velocity with respect to the $O(i)$'s

$$v^2(i) = \frac{2GM}{r(i)} \tag{6.3}$$

and, up to a rotation, coincides asymptotically with the observers at rest at spatial infinity in the condition of vanishing gravitational field.

We shall restrict to a weak-field condition so that the corrections to the classical theory can be evaluated by considering circular orbits of radius $r(i)$. The order of the $O(i)$'s is such that $r(i) < r(i+1)$ and we assume $\frac{2GM}{c^2 r(1)} \ll 1$. For instance, for the sun, the value $r(1) \sim R_{\rm sun}$ gives $\frac{2GM}{c^2 r(1)} \sim 4 \cdot 10^{-6}$.

To leading order, the relation between the $O(i)$'s and K_o is a Lorentz-transformation with velocity (6.3) so that the set of flat metrics

$$ds^2(i) = c^2 dt^2(i) - dr^2(i) - dl^2 \tag{6.4}$$

implies

$$ds_o^2 = c^2 dt^2(i)\left[1 - \frac{2GM}{c^2 r(i)}\right] - \frac{dr^2(i)}{1 - \frac{2GM}{c^2 r(i)}} - dl^2 \tag{6.5}$$

Namely, if K_o wants to measure the space-time interval between two events with radial components infinitesimally close to $r = r(i)$, by using the space-time units of the corresponding $O(i)$, obtains Eq.(6.5). Notice, however, that the relation between the $O(i)$'s and K_o is a Lorentz transformation. Therefore, Eqs.(6.4) and (6.5) do *not* refer to the same pair of events.

To address the problem of perihelia we observe that the Keplerian orbits are computed by assuming the validity of the Galilean tranformation $t' = t$, thus giving an absolute meaning to angular velocities. This is no longer true since we know that $\frac{dl}{dt(i)} \neq \frac{dl}{dt_o}$. The difference can be treated as a small perturbation to the Keplerian orbit

$$\delta U = -\frac{L^2}{2mr^2}\frac{2GM}{c^2 r} \equiv \frac{\gamma}{r^3} \tag{6.6}$$

where L and m denote the angular momentum and the mass of the body in the bound orbit. To first-order in δU, we can use the result of [46]

$$\Delta\varphi = -\frac{6\pi\gamma}{GMmp^2} \tag{6.7}$$

where $p^2 \equiv a^2(1-e^2)^2$, a and e being the parameters of the bound orbit. Therefore, replacing the value of the angular momentum

$$L^2 = \frac{4m^2\pi^2}{T^2}a^4(1-e^2) \tag{6.8}$$

we get the final expression

$$\Delta\varphi = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)} \tag{6.9}$$

that, indeed, is the same expression as computed in general relativity. For this reason, by following the original suggestion by Schiff [5], we conclude that, to the present level of accuracy, all classical experimental tests of general relativity would be fulfilled in *any* theory that incorporates the Equivalence Principle.

This result reflects the very general nature of the infinitesimal transformation to the rest frame of a freely falling elevator. For instance, this can also be implemented with a conformal (acceleration) transformation of space and time and a tranformation of mass [47]

$$m \to m[1 + \tilde{\phi}(x)] \tag{6.10}$$

that includes the gravitational energy. Eq.(6.10) would be extremely natural in an approach where the gravitational potential is due to a long-range fluctuation of the shifted Higgs field and suggests some considerations on the important physical meaning of conformal transformations and the basic assumption of a riemannian space-time in general relativity.

Suppose we follow the basic idea underlying the Equivalence Principle, namely a sequence of infinitesimal acceleration tranformations to remove the effects of a weak gravitational field. Which is the final space-time structure ? This is an interesting question since *we are* in a freely falling reference frame and, therefore, we want to understand the properties of space-time also from this point of view. Since we start from Minkowski space-time, an acceleration transformation is naturally defined as an element of the 15-parameter group C_o in terms of a constant 4-vector κ_μ

$$x'_\mu = \frac{x_\mu + \kappa_\mu x^2}{1 + 2(\kappa \cdot x) + \kappa^2 x^2} \tag{6.11}$$

This is singular at some values of x_μ. For instance, in the case $\kappa_\mu = (0; 0, 0, \frac{g}{2c^2})$ the singularity occurs for $t = \frac{2c}{g}|1 - \frac{g}{2c^2}z|$. The point is that only the product of the Poincare' group with scaling tranformations, the Weyl group, operate *globally* on $\mathcal{M}_o$ [48]. Thus, if X is the generator of an infinitesimal acceleration transformation (that is not in the Weyl group) and p is a given point of $\mathcal{M}_o$, $e^{(sX)}p$ is well defined for a sufficiently small value of s (whose range depends on p) but for no value of s is $e^{(sX)}$ defined throughout $\mathcal{M}_o$.

Since the local gravitational field is not a constant, i.e. $\kappa_\mu = \kappa_\mu(x)$, the singularity has not a real physical meaning but represents a signal that the flat Minkowski space-time is carried out of itself into a larger covering space with typical local curvature $\sim \frac{c^2}{|g(x)|}$. In general, conformal transformations lead to a covering space that is *not* a riemannian space but a Weyl space [47]. The same problem, in the framework of induced-gravity theories [1, 2, 3], means that one should compute the effective lagrangian in a general background metric where besides a symmetric tensor $g_{\mu\nu}$, there is a vector $\kappa_\mu(x)$ at each point so that the Weyl connection is different from the Christoffel symbol.

The crucial point, however, is that κ_μ is a gradient, since the gravitational acceleration field is $\kappa_\mu = \partial_\mu \tilde{\phi}$, so that the Weyl space is *equivalent* to a riemannian space [47]. In this case, i.e. when the origin of the gravitational acceleration is due to a scalar potential, the basic assumption of a riemannian space becomes consistent with the intuitive indications obtained from acceleration transformations. Therefore, the weak-field effective lagrangian in a background space with arbitrary metric tensor $g_{\mu\nu}$ and Weyl connection $\kappa_\mu = \partial_\mu \tilde{\phi}$ will always reproduce Einstein field equations, for a suitable choice of the energy-momentum

tensor. Here 'suitable' means that the Minkowski-space $T_{\mu\nu}$ is, in general, modified for $\tilde{\phi}$-dependent terms and that the definition of the energy-momentum tensor relevant for Einstein field equations does not contain any *large* cosmological term from spontaneous symmetry breaking. The latter is an obvious consequence of generating the theory in curved space-time with a series of conformal transformations from flat space.

On the other hand, quite independently of conformal transformations, a very intuitive argument to understand why a large term as $g_{\mu\nu}\mathcal{E}(n_v)$ cannot enter Einstein field equations is the following. In Einstein's original picture, all forms of energy and matter contribute to the space-time curvature. However, phions, although possessing an inertial mass, have no 'gravitational mass' and cannot generate any curvature. This statement represents the geometrical counterpart of a non-Lorentz covariant energy spectrum $\tilde{E}(k)$ for $k \to 0$, responsible for the instantaneous nature of the gravitational force. From this point of view, the scalar condensate is, for gravitational phenomena, a real preferred frame and can be considered the quantum realization of the old-fashioned *weightless* aether. Notice that the idea of a *quantum* aether, of the type that can be generated from the ground state of a quantum field theory, was considered by Dirac long time ago [49]. In this case, Dirac's aether velocity field u_μ coincides with κ_μ and represents the four-dimensional analogue of the superfluid velocity flow (5.14).

Finally, there seems to be good experimental evidence for a *small* cosmological term [50]. Its typical size is comparable with the ordinary-matter contribution and the combination of the two effects gives precisely a spatially-flat universe. Outside of our framework, i.e. without giving a special role to conformally-flat space, it would be very hard to understand this result. On the other hand, by following the picture where the space-time curvature is generated by a sequence of acceleration transformations on Minkowski space, this small cosmological term is another way to introduce the cosmic acceleration field g_o Eqs.(5.19)-(5.21) associated with the expansion of the universe.

If, in the end, it will turn out that the 'preferred' [51] metric structure $\bar{g}_{\mu\nu}$ of the universe requires the introduction of a cosmological term, this will provide an effective graviton mass term in Einstein weak-field equations for $g_{\mu\nu} \sim \bar{g}_{\mu\nu} + h_{\mu\nu}$. At the same time, if this has to reflect the dynamical origin of gravity, it may correspond to a superluminal propagation of gravitational 'waves' [52]. As anticipated in Sect.2, our point of view is closely related to the violations of causality in general relativity with a cosmological constant [12].

7. Summary and concluding remarks

In this paper we have presented a simple physical picture where Newtonian gravity arises as a long-wavelength excitation of the scalar condensate inducing spontaneous symmetry breaking. Our proposal represents the most natural interpretation of an important phenomenon that has been missed so far: the gap-less mode of the (singlet) Higgs field. Its existence is a direct consequence of Bose-Einstein condensation and has to be taken into account *anyway* (namely, how can we interpret it without gravity ?).

We emphasize that our main result in Eq.(5.4) depends only on the Riemann-Lebesgue theorem on Fourier transforms [35] and two very general properties of the excitation spectrum. Namely, the 'diluteness' condition Eq.(4.16) (that leads to the 'sound-wave' shape in Eqs.(4.11) and (4.13) for $k \to 0$) and the Lorentz-covariance for large k (that leads to Eq.(4.10)). These features are expected to occur in *any* description of spontaneous symmetry breaking in terms of a weakly coupled Bose field and depend on the weakly first-order nature of the phase transition. This occurs for a very small but non-vanishing value of the phion mass m so that there is a non-relativistic regime $k \ll m$ where the scalar condensate responds with phase-coherence. As discussed in detail in Sects.3 and 4, this result reflects the existence of an ultimate ultraviolet cutoff responsible for the deviations from an exactly Lorentz-covariant spectrum for $k \to 0$ in the broken phase.

A more complete description of gravitational phenomena requires the detailed form of the energy spectrum $\tilde{E}(k)$ and, in particular, the precise knowledge of the phion mass m. Deviations from the Newton potential are expected at typical distances $r \sim 1/m$ and could, eventually, be detected in the next generation of precise 'fifth-force' experiments [53]. We emphasize that these deviations from the $1/r$ law can change the description of the gravitational clustering of matter and are essential to understand whether (or not) gravity remains in a weak-field regime for a large gravitational mass. In our description, where gravity is the remnant of an almost 'trivial' theory, this would be the most natural conclusion.

Finally, our description of the origin of gravity from the scalar condensate leads to the simple hydrodynamical picture outlined in Eqs.(5.13)-(5.18). This provides a clue to the peculiar modification of Newtonian gravity [38] that solves the experimental mass discrepancy in many galactic systems and represents a completely new approach to the problem of dark matter.

We emphasize that, our description of gravity, although predicting new phenomena, is not logically in contradiction with general relativity, at least in an obvious way. This

can be understood by realizing that two main outcomes of our picture, the Equivalence Principle and the 'Mach's Principle' view of inertia were two basic ingredients at the origin of Einstein theory [44]. For this reason, the classical tests of general relativity in weak gravitational field are fulfilled as in *any* theory incorporating the Equivalence Principle. Further, the special role of infinitesimal conformal transformations to implement the transition to the rest-frame of a freely falling elevator suggests that Einstein equations may represent the effective weak-field approximation of a theory generated from flat space with a sequence of conformal transformations. This can easily explain the absence of a *large* cosmological term in Einstein field equations.

We stress that the apparently 'trivial' nature of $\lambda\Phi^4$ theories in four space-time dimensions should not induce to overlook the possibility that gravity can arise as a gap-less mode of the Higgs field. Indeed, our description is only possible if one assumes the existence of an ultimate ultraviolet cutoff so that the natural formulation of the theory is on the lattice and 'triviality' is never complete. In this case, however, the existence of a non-trivial infrared behaviour in the broken phase is not surprising due to the equivalence of low-temperature Ising models with highly non-local membrane models on the dual lattice [54] whose continuous limit is some version of the Kalb-Ramond [55] model. Thus, in the end, a Higgs-like description of gravity may turn out to be equivalent, at some scale, to a Feynman-Wheeler theory of *strings*, as electromagnetism for point particles [56]. At the same time, the basic idea that one deals with the *same* theory should allow to replace a description with its 'dual' picture when better suited to provide an intuitive physical insight.

To conclude, we want to mention another implication of the scalar condensate at the astronomical level. As anticipated, our picture provides a natural solution of the so-called 'hierarchy-problem'. This depends on the *infinitesimally weak* first-order nature of the phase transition: in units of the Fermi scale, $M_h \sim G_F^{-1/2}$, the Planck scale $G^{-1/2}$ would diverge for a vanishing phion mass m. These scales are hierarchically related through the large number $\sqrt{\eta} \sim 10^{17}$ which is the only manifestation of an ultimate ultraviolet cutoff. In this sense, spontaneous symmetry breaking in $(\lambda\Phi^4)_4$ theory represents an approximately scale-invariant phenomenon and it is conceivable that powers of the 'replica-factor' 10^{17} will further show up in a natural way (notice that in our picture the factor $\eta^{1/6} = \mathcal{O}(10^5)$ also appears, see Sect.4 and fig.4 of ref.[14]). For this reason, after the scales $m \sim 10^{-5}$ eV and $\sqrt{\eta}m \sim 10^2$ GeV, we would expect the scale $\frac{m}{\sqrt{\eta}} \sim 10^{-22}$ eV to

play a role [57] at the astronomical level in connection with a length

$$l \sim \frac{\sqrt{\eta}}{m} \sim 10^{17} \text{ cm.} \tag{7.1}$$

To this end, we observe that, in the field of a large mass as the sun, the 'MOND' regime [38] mentioned in Sect.5 corresponds precisely to the length scale l , namely

$$g_o \sim \frac{GM_{\text{sun}}}{l^2} \sim 10^{-8} \text{ cm sec}^{-2} \tag{7.2}$$

As anticipated, in our picture this also corresponds to a situation where the phion density n differs substantially from its average value (5.16) related to the Newton constant and the Poisson equation (5.12) and the equivalent condition of a constant-density potential flow Eqs.(5.14)-(5.15) cease to be valid. This means that beyond l we enter in a 'G-variable' theory [58].

Even at distances $r \ll l$ the effects of g_o may, however, be observable if there is a Hubble flow over solar-system scales. As discussed in Sect.5, in fact, there would be a cosmological velocity field $\langle \mathbf{u}_s \rangle$ associated with the continuous creation of phions in an expanding universe. The basic idea of a Hubble flow over small scales has been recently re-proposed [59, 60] in connection with the observation by Anderson *et al* [61] of an anomalous acceleration $g_{\text{anom}} \sim g_o$ from the Pioneer data at the border of the solar system ($r \sim 10^{15}$ cm). Since this interpretation has been shown [60] to be consistent with planetary and even geological data, (the unexplained part of) the Pioneer anomaly may represent the first evidence of a fundamental phenomenon related to the true dynamical origin of inertia and gravity.

Finally, at very large distances $r >> l$ one has to use the more general Eqs.(5.17)-(5.18) to determine the gravitational field for a given distribution of matter $\sigma(x)$. In this case, the properties of galactic systems require strong departures from Newtonian dynamics and the gravitational acceleration is found $g \sim 1/r$ rather than $1/r^2$ [38]. A full understanding of this result requires to *predict* the form of the function f in (5.18) by evaluating those residual self-interaction effects in the scalar condensate responsible for the transition from the constant-density regime described by Eqs.(5.12), (5.15) to the more general Eqs.(5.17), (5.18). The idea of explaining the mass discrepancy without any form of 'dark matter' is not unconceivable. Indeed, the typical values for the visible masses of gravitating matter

$$M_{\text{galaxy}} \sim 10^{11} M_{\text{sun}} \sim \eta^{1/3} M_{\text{sun}} \tag{7.3}$$

and

$$\rho_{\text{galaxy}} \sim 10^4 \text{parsec} \sim \eta^{1/6} l \tag{7.4}$$

lead to the same value as in Eq.(7.2)

$$g_o \sim \frac{GM_{\mathrm{galaxy}}}{\rho^2_{\mathrm{galaxy}}} \sim 10^{-8}\ \mathrm{cm\ sec}^{-2} \tag{7.5}$$

Similarly, the values (U= visible universe)

$$M_{\mathrm{U}} \sim \eta^{1/3} M_{\mathrm{galaxy}} \sim \eta^{2/3} M_{\mathrm{sun}} \tag{7.6}$$

and

$$\rho_{\mathrm{U}} \sim 10^{10}\ \text{light years} \sim \eta^{1/3} l \tag{7.7}$$

give again

$$g_o \sim \frac{GM_{\mathrm{U}}}{\rho^2_{\mathrm{U}}} \sim 10^{-8}\ \mathrm{cm\ sec}^{-2} \tag{7.8}$$

This suggests that the properties of the scalar condensate can indeed play an essential role to understand the steps in the cosmological hierarchy [62, 63].

Note added in proof

After finishing this paper I have become aware of a recent preprint by F. Ferrer and J. A. Grifols, *Effects of Bose-Einstein condensation on forces among bodies sitting in a boson heat bath*, hep-ph/0001185. In this paper, the presence of a long range $1/r$ potential in connection with the Bose Einstein condensation of a massive scalar field is also pointed out. I thank P. M. Stevenson for this information.

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than the sound velocity [34]. Admittedly, we have not worked out any quantitative estimate of this possible reduction effect.

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Figure Caption

Fig.1 A qualitative pictorial representation of the energy spectrum $\tilde{E}(k)$. The actual relative sizes are such that the continuous matching inside the shaded blob is at a value $k \sim m = \mathcal{O}(10^{-17})$ in units of M_h.